The University of Georgia
Department of Mathematics and Science Education
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Final Assignment (Part Two)

by

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Exploration Topic:

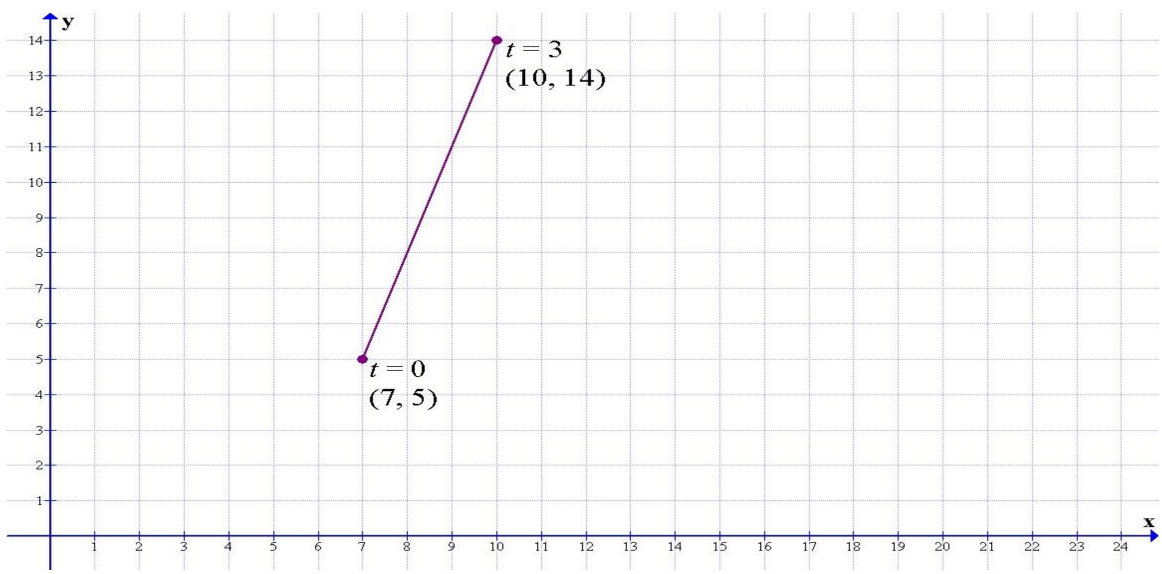
a) Construct parametric equation of a line segment through $(7,5)$ with slope of 3.
b) Graph the line segment with the equation.
c) Explore ways of choosing endpoints such that the two distances from $(7,5)$ are 2 units and 3 units.

a) There are many ways to write the parametric equations of a line segment for given slope and point. In this case, I will use the coordinates of the point given as the constant terms in the parametric equation. Hence, 7 would be the constant for $x(t)$ and 5 is the constant for $y(t)$. Now, for slope 3, for each unit that x-coordinate moves, the y-coordinate moves 3 units. So we can form the parametric equations for the case given by

$$x(t) = t + 7$$
$$y(t) = 3t + 5$$

Note: these parametric equations initial point (t=0) is the point given $(7,5)$.

b)



c) Now, let's find the values of t such that the distance from $(x(t), y(t))$ to $(7,5)$ is the desired value.

For general equation, using the distance formula, we get:

$$r = \sqrt{(x(t) - 7)^2 + (y(t) - 5)^2}$$

$$r = \sqrt{(t + 7 - 7)^2 + (3t + 5 - 5)^2}$$

$$r = \sqrt{t^2 + (3t)^2}$$

$$r = \sqrt{10t^2}$$

$$r = t\sqrt{10}$$

$$t = \frac{r}{\sqrt{10}}$$

For the first endpoint, it is located 2 units from$(7,5)$. So, using the above formula for $r = 2$, we get the t value

$$t = \frac{2}{\sqrt{10}}$$

Now, using this value in the parametric equation would give us an endpoint in the segment line 2 units from(7,5). We can also substitute the additive inverse of the value to get the same distance but in opposite direction. The later will be done for the case when $r = 3$.

Replacing the value of $t$found in the parametric equations, we get:

$$x\left(\frac{2}{\sqrt{10}}\right) = \frac{2}{\sqrt{10}} + 7 = \frac{\sqrt{10}}{5} + 7 = \frac{\sqrt{10} + 35}{5}$$

$$y\left(\frac{2}{\sqrt{10}}\right) = 3\left(\frac{2}{\sqrt{10}}\right) + 5 = \frac{6}{\sqrt{10}} + 5 = \frac{3\sqrt{10}}{5} + 5 = \frac{3\sqrt{10} + 25}{5}$$

Hence,

$$\left(\frac{\sqrt{10} + 35}{5}, \frac{3\sqrt{10} + 25}{5}\right)$$

For the second case$(r = 3)$, replacing the value of t found in the parametric equations, we get:

$$x\left(-\frac{3}{\sqrt{10}}\right) = -\frac{3}{\sqrt{10}} + 7 = -\frac{3\sqrt{10}}{10} + 7 = \frac{-3\sqrt{10} + 70}{10}$$

$$y\left(-\frac{3}{\sqrt{10}}\right) = 3\left(-\frac{3}{\sqrt{10}}\right) + 5 = -\frac{9}{\sqrt{10}} + 5 = -\frac{9\sqrt{10}}{10} + 5 = \frac{-9\sqrt{10} + 50}{10}$$

Hence,

$$(\frac{-3\sqrt{10} + 70}{10}, \frac{-9\sqrt{10} + 50}{10})$$